الميكانيك الهندسي

المرحلة الاولى

قسم ميكانيك القدرة / فرع السيارات

اعداد:

صولة طه حامد

مدرس مساعد

Reference:

1- Engineering mechanics by F. I Singer.
2- Strength of materials by Singer.

Basic units:

Quantity	Name	Symbol
Length (L)	Meter	M
Mass (m)	Kilogram	Kg
Time (t)	Second	Sec
Force (f)	Newton	N
Acceleration (a)	meter/sec^2	m/sec^2
Velocity (v)	meter/sec	m/se

Engineering mechanics

Static

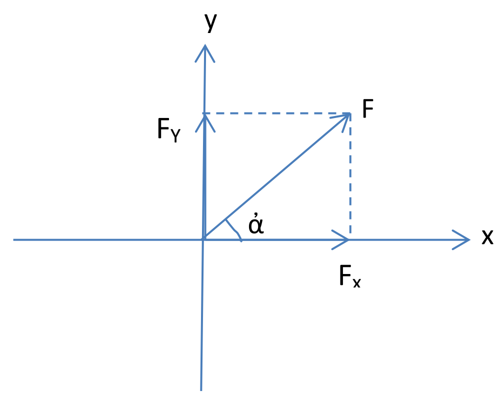
Is that branch of eng. mech. Which deals with the force and their effects while acting upon the bodies at rest.

Dynamic

Is that branch of eng. mech. Which deals with the force and their effects while acting upon the bodies in motion.

Values of trigonometric function for some typical angles:

Angle	0^0	30^0	45^0	60^0	90^0
Sin	0	1/2	$1/2^{1/2}$	$3^{½}/2$	1
Cos	1	$3^{½}/2$	1/2½	1/2	0
Tan	0	$1/3^{1/2}$	1	$3^{1/2}$	$\dot{\alpha}$



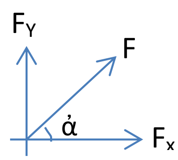
Analysis of force

$F_x = F * \cos \dot{\alpha}$

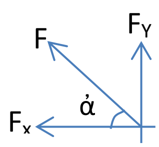
$F_y = F * \sin \dot{\alpha}$

Direction of force (F):

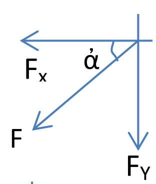
1- Up to the right:



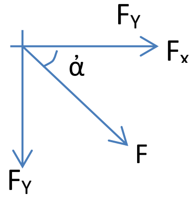
2- Up to the left:



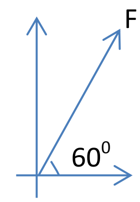
3- Down to the left:



4- Down to the right:



Ex1: find the x and y component of the forces 450 N with 60^0 up to the right as shown:



Fx = F.cos60

= 450 * 0.5

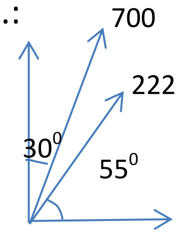
= 225 N

Fy = F.sin 60

= 450 * 0.866

= 389.7 N

Ex2: determine (x , y) component for each forces shown in fig.:



<u>solution:</u>

$F_1 x = F_1 \cos 55$

= 222 * 0.57

= 127 N

$F_1 y = F_1 \sin 55$

= 222 * 0.81

= 182 N

$F_2 x = F_2 \cos 70$

= 700* 0.34

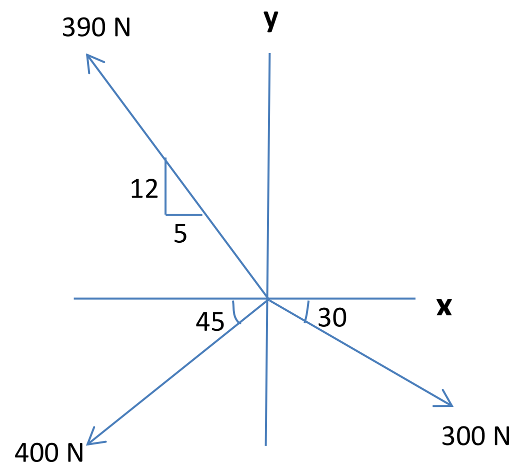
= 238 N

$F_2 y = F_2 \sin 70$

= 700 * 0.94

= 657 N

Ex_3 : determine the x , y component for each force shown in figure:



$F_1x = F1 \cos30$

$= 300 * 0.866$

$= 252$ N

$F_1y = F_1 \sin 30$

$= 300 * 0.5$

$F_2x = F_2 \cos45$

$= 400 * 0.707$

$\tan\Theta = 12/5 = 2.4$, $\Theta = 67.3^0$

$F_3x = F3\cos67.3 = 390 * 0.8 = 19.8$ N

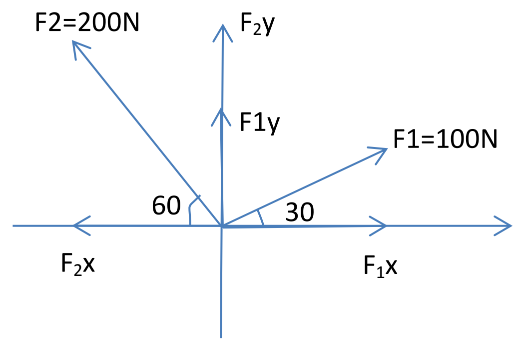
$F_3y = F_3\sin67.3 = 390 * 0.92 = 359$ N

Resultant of forces:

Resultant of forces by summation of component method:

1- Find the component of all forces>
2- Add the component with x- axis $\sum Fx$.
3- Add the component with y- axis $\sum Fy$.
4- Find the resultant by $R = \sqrt{((\sum Fx)^2 + (\sum Fy)^2)}$
5- Find the direction of the resultant:
   $\Theta = \tan^{-1} \sum Fy/\sum Fx$

Ex1: find the resultant of the forces system shown in fig.



F_1x = F1 cos30

= 100 *0.866

= 86.6 N

F_1y= F_1sin30

= 100 * 0.5

F_2x = F_2cos60

= 200 * 0.5

100 N

F_2y = F_2 sin60

=200 *0.866

= 173.2 N

∑Fx = F_1x + F_2x

= 86.6-100 = - 13.4 N

∑Fy = F_1y +F_2y

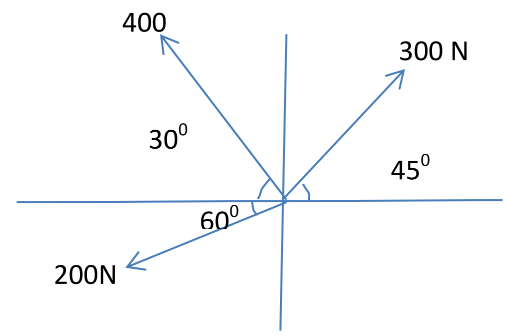
= 50 +173.2 = 223.2 N

R = $\sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R = $\sqrt{((-13.4)^2+ (223.4)^2)}$

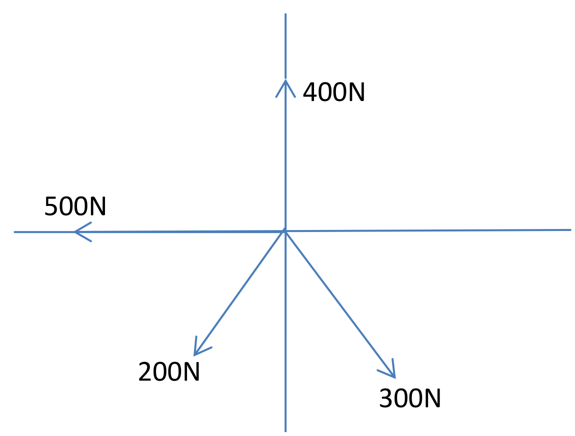
tan Ө = ∑Fy /∑Fx

Ex2: find the resultant of the force system shown in fig. :



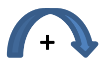
Ex3: determine the resultant of the forces system

shown in fig. :

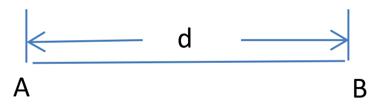


Moment:

The moment of any forces its measure of its ability to produce turning or twisting about any point or axis.

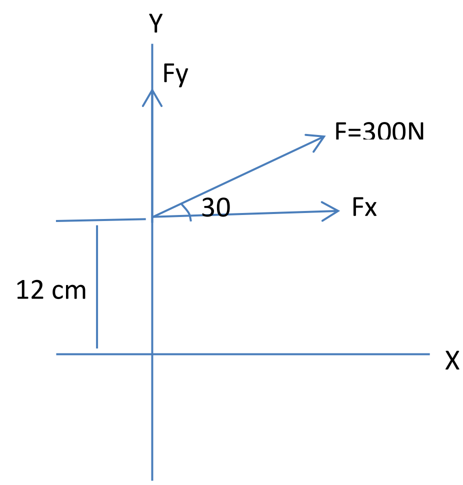
Clockwise 

Counter clockwise 



Ex1: find the moment of the force shown in fig. about the origin:

Sol.



Fx = F. cos30

= 300 * 0.866

= 259.8 N

Fy = F sin30

= 300 *0.5

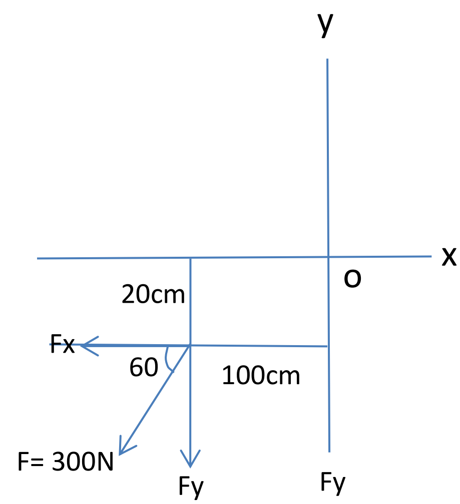
M =F * d

M = Fx * 12 + 0

= 259.8 8 12

= 3117.6 N

Ex.2: find the moment of the force 300N shown in fig. about the origin:



Fx = F cos60

= 300 * 0.5

=160 N

Fy = F sin60

= 300*0.866

= 259.8N

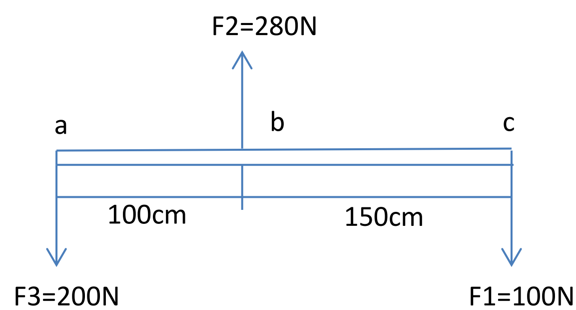
M = ∑ F.d

= Fx * 20 + Fy * 100

= 150 *20 – 259.8 *100

= -229980 N .cm

Ex3: find the moment of the force shown in fig. about the points (a , b , c):



Sol.

Ma= F_1 * 250 – F_2 *100

= 100 * 250 – 2800 *100

= 255000 N.cm

Mb = F_1 * 150 – F_3 *100

= 5000 N.cm

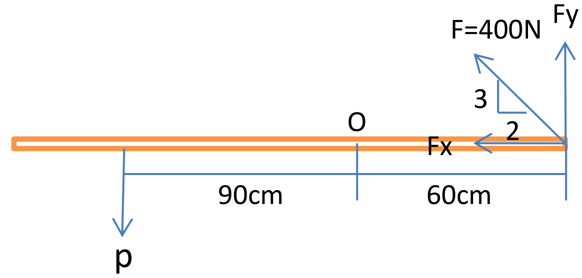
Mc= F_2 * 150 – F_3 *250

= 2800 * 150 – 200 * 250

= 370000 N.cm

Ex4: find (p) if the moment about the point (o) shown in fig. equal (-1250 N.m):

Sol.:



$\tan\theta = 3/2 = 1.5$

$\theta = \tan^{-1} 1.5 = 56.3^0$

MFx = 0

Fy = Fsin56.3

= 400 * 0.831

= 332.4

Mo = -1250 = -p * 0.9- Fy * 0.6

= -0.9p – 332.4 * 0.6

= -1250

-0.9p- 199.44 = -1250

-0.9p = 199.44-1250

-0.9p= -1050.56

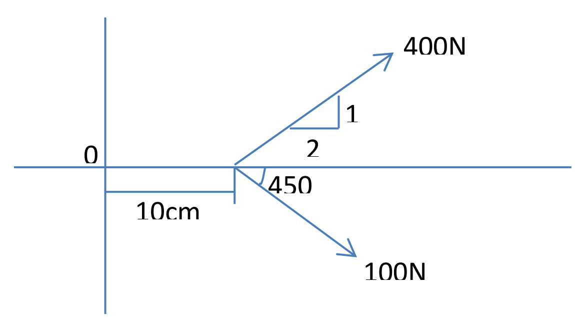
P = 1167.288 N

Ex3: find the moment of the force shown in fig. about the point (O):

$\tan \theta = ½$, $\theta = 26.5^0$

Mo = $-F_1y$ 10 + F_2y* 10

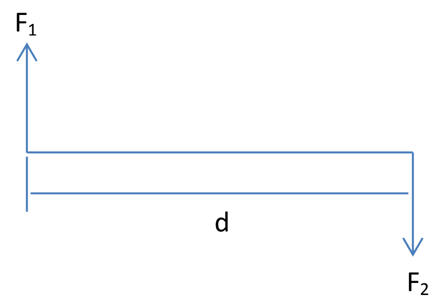
= -16900 N.m



The couple:

The coupe is teo forcs acting on the body:

1-have the same magnitude $F_1 = F_2$.



2- parallel each other.

3-the resultant equal to zero R=0.

4-have the same moment.

Mc = F_1 *d

Ex1: find the resultant of the forces F1 and F2 shown in fig. also find the couple produced by them:

Sol.

R =0

Mc = -100 * 75

= -7500 N.m

Ex2: for beam shown in fig. find the moment with respect to points (A,B,C,D,E) also prove that the system forces is couple:

M_A = 10 * 2 - 20 * 5 – 30 * 6 + 40 * 8

= 60 N .m

M_B = -20 *3 -30 *4 + 40 *6

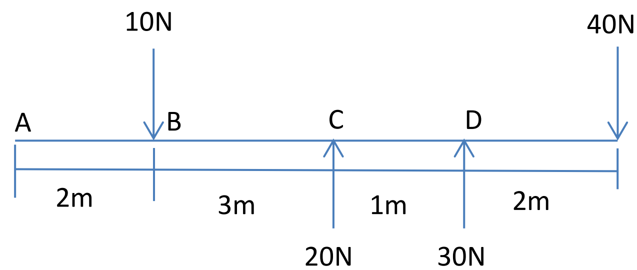
= 60 N .m

Mc = -10 * 3 – 30 *1 + 40 * 3

= 60 N .m

M_D = -10 *4 +20*1 +40 *2

= 60 N .m



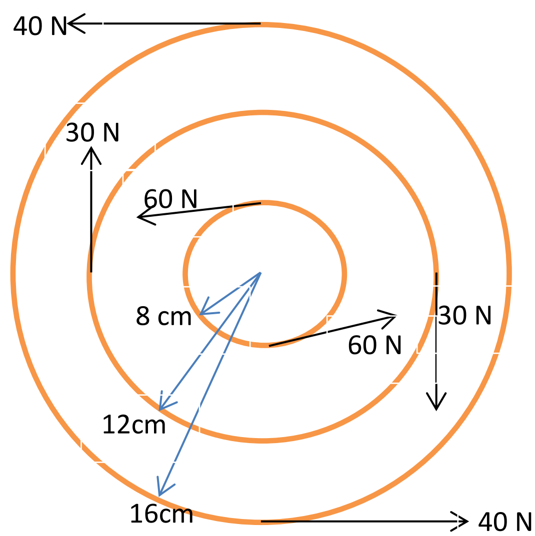
M_E =-10 *6 +20 *3 +30 *2

=60 N .m

R = -10 +20 +30 – 40

= 0

Ex3: the three step pulley shown in fig. subjected to the given couples compute the value of resultant also determine the force acting on the arm of the middle pulley that required place the given system:



Mc = (-40 * 16) + (30 *12) - (60 * 8)

= - 640 + 360 – 480

= - 760N.m

Mc = F. d

-760 = F * 12

F = -63.3 N

Equilibrium of force system:

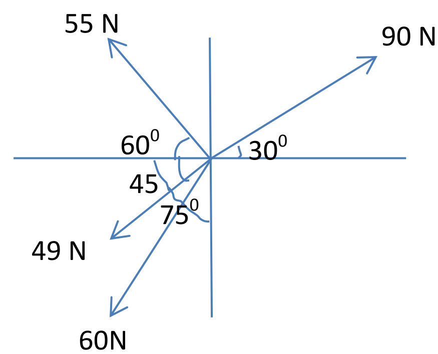
Equilibrium is the condition where the resultant of the system of force is zero:

R = 0

Equilibrium of concurrent force system:

1- Rx = 0
2- Ry = 0
3- $R = \sqrt{(R^2x + R^2y)} = 0$

Ex.1: prove that the forces shown in fig. are in equilibrium:



Sol.:

$\sum Rx = 90 \cos 30 - 55 \cos 60 - 49 \cos 45$

$= -60 \cos 75$

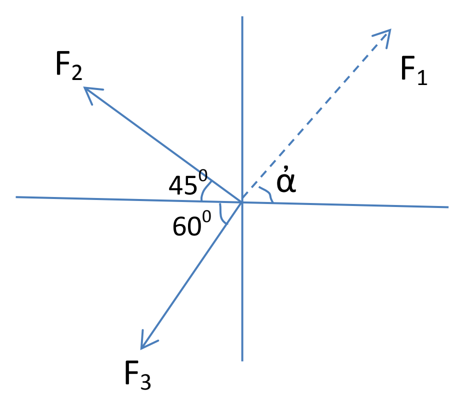
$= 0$

$\sum Ry = 90 \sin 30 + 55 \sin 60 - 49 \sin 45$

$= 0$

So the force are in equi.

Ex.2: determine the force F_1 required to make the system shown bellow balanced:



$F_2 = 134$ N

$F_3 = 82$ N

$F_{1X} - F_{2X} - F_{3X} = 0$

$F_{1X} = F_{2X}$ COS45 + F3 COS60

$= 135.7$ N

$F_{1y} = F_{3y} - F_{2y} = \_\ 23.7$ N

$F_1 = \sqrt{((F_{1X})^2 + (F_{1y})^2)} = 137.75$ N

$\tan ά = F_{1y}/F_{1x} = F_1 = \_\ 9.9^0$

Equilibrium of non-concurrent force system:

$R_X = 0$

$Ry = 0$ — The force are equilibrium

$\sum M_O = 0$

E_{X1}: is the body shown in figure balanced?

$F_1 = 200$ N , $F_2 = 228$ N , $F_3 = 100$ N , $F_4 = 191.4$N

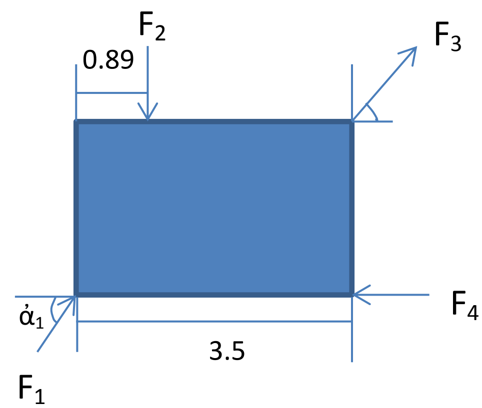
$\dot{\alpha}_1 = 45^0$, $\dot{\alpha}_2 = 60^0$

Sol.

$RX = F_{1X} - F_4 + F_{3X}$

$= F_1\cos45 - F_4 + F_3\cos60$

$0 = 200\cos45 - 191.4 + 100\cos60$



$R_y = F_{1y} + F_{3Y} - F_2$

$= F_1\sin\dot{\alpha}_1 + F3\sin\dot{\alpha}_3 - 228 = 0$

$M = - F_2 * 0.89 + F_{3y} * 3.5 - F_{3x} * 2$

$= 0$ the body blanced

Ex.2 : for the beam shown bellow find F_A , $\dot{\alpha}_A$ to make it balanced:

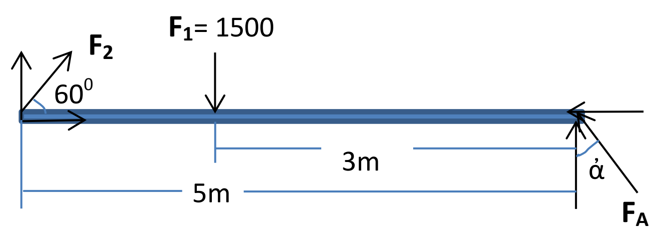
Sol.

$\sum Fx = 0$

Sol.

$\sum Fx = 0$

$F_2x - F_{Ax} = 0$



$F_2x = F_Ax$ .... (1)

$\sum Fy = 0$

$F_Ay = F_1 – F_2y$

$= 1500 – F_2y$ ....(2)

$\sum MA = 0$

$F_1 * 3 – F_2y * 5 = 0$

$F * 3 – F_2y * 5 = 0$

$F_2y = 3F_1/5$

$= 1500 *3/5$

$= 900$ N

Sub in (2)

$F_Ay = 1500 – 900$

$F_Ay = 600$ N

$\tan 60 = F_2y/F_2x$

$F_2x = F_2y/\tan 60$

$= 900/1.732$

$= 520$N

$F_A = F_2x =520$ N

$F_A = \sqrt{((520)^2 + (600)^2)}$

$= 794$N

$\dot{\alpha}_A = \tan^{-1}(F_Ax/F_Ay)$

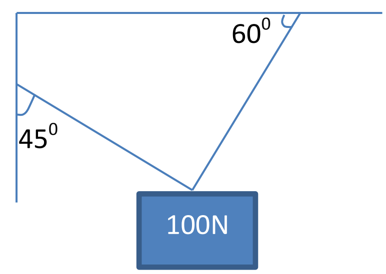
$F_2 =\sqrt{((F_2x)^2 + (F_2y)^2}$

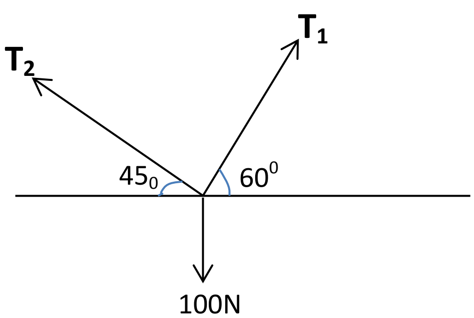
$= \sqrt{((520)^2 + (900)^2)}$

$= 1039N$

Equilibrium of concurrent force system:

Ex: a body of weight (100N) is hanging by two cable as shown in fig. find the tension in each cable:





Sol.

$Rx = \sum Fx - 0$

$T1x - T2x = 0$

$T_1\cos 60 - T_2\cos 45 = 0$

$0.5T_1 - 0.707T_2 = 0$

$\sum Fy = 0$

$T_1y + T_2\sin 45 = 0$

$0.866T_1 + 0.707T_2 = 100$

$0.5T_1 - 0.707T_2 = 0$

$1.366\ T_1 = 100$

$T_1 = 100/1.366N$

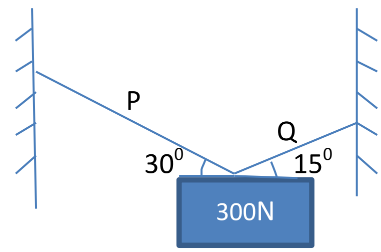
$= 73.2N$

Sub in (1):

$0.5(73.2) - 0.707T_2 = 0$

$T_2 = 51.76N$

Ex2: find the tension P and Q acting on the rope shown in fig. :



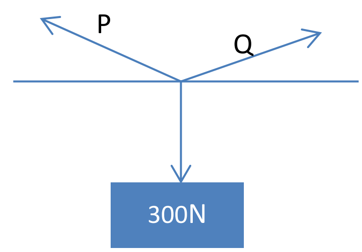
Sol.

$\sum Fx = 0$

Qx – Px = 0

Qcos15 – Pcos30 = 0

0.965Q – 0.866P = 0 .... (1)



$\sum Fy = 0$

Qy + Py – 300 = 0

Qsin15 +Psin30 = 300

0.253Q+0.5P = 300 .... (2)

- 0.705P = - 290

P = 290/0705

= 411

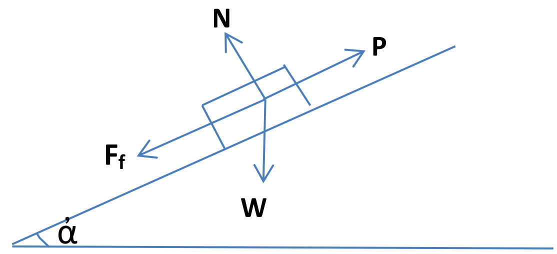
Sub in (1):

0.965Q – 0.866 * 411 = 0

Q = 356/0.965

= 369N

Friction:



$F_f = \mu.N$

F_f = friction force

μ = coefficient of friction

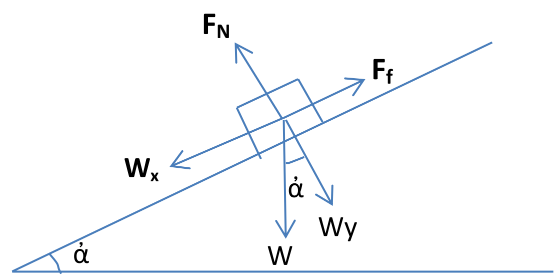
N = normal force

Friction force is always opposite to the direction of motion.

Angle of friction:

Ex1: find the angle at which the body will be move down word:

Sol.



$\sum Fx = 0$

$F_f – W_x = 0$

$Ff = Wx$

$= W\sin\dot{\alpha}$ ..... (1)

$\sum Fy = 0$

$F_N - Wy = 0$

$F_N = W_y$

$= W\cos\dot{\alpha}$

$F_f = \mu.F_N$

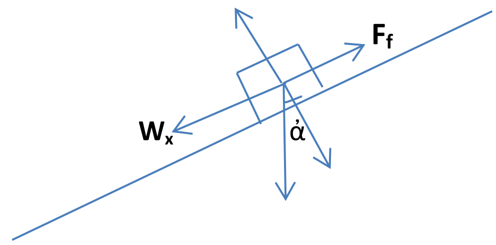
$\mu = F_{f/} F_N$

$= mg\sin\dot{\alpha}/mg\cos\dot{\alpha}$

$\dot{\alpha} = \tan^{-1}\mu$

Ex2.: a 200N black is at rest on a 30^0 incline the coefficient of friction (0.2) compute the value of a horizontal force **P** that will cause motion up the incline:

$\sum Fy = 0$



FN – Py – Wy = 0

$F_N = P\sin30 + 200\cos30$

$= 0.5P + 173.2$ ..... (1)

$Ff = \mu . F_N$

$= 0.2(0.5P + 173.2)$

$= 34.64 + 0.1P$

$\sum F_X = 0$

$P_X - Ff - W_X = 0$

$P_X = F_f + Wx$

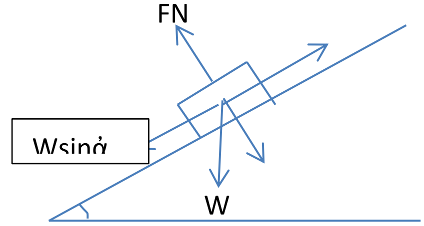
$P\cos30 = (34.64 + 0.1p) + 200\sin30$

P = 176N

Ex3.: abody as shown in fig. moving up word at constant speed on a surface of an angle $\dot{\alpha} = 30^0$. find its mass if F = 80N.

Sol.

$\sum Fx = 0$



$= F - F_f - Wx$

$F = F_f + W\sin\dot{\alpha}$ .... (1)

$= F_f + mg.\sin\dot{\alpha}$

$F_y = 0 = F_N - W\cos\alpha$

$F_N = mg.\cos\alpha$ .... (2)

$F_f = \mu.F_N$ .... (3)

$Ff = \mu.mg.\cos\alpha$

$F = \mu.mg.\cos\alpha + mg.\sin\alpha$

$F = mg.(\mu\cos\alpha + \sin\alpha)$

$m = F/g(\mu\cos\alpha + \sin\alpha)$

$= 80/9.8(0.2\cos30 + \sin30)$

$= 12.12kg$

Ex.4: what weight W is necessary to start the system of blocks shown in fig. moving to the right? The coefficient of friction is 0.1 and the pulleys are assumed to be frictionless.

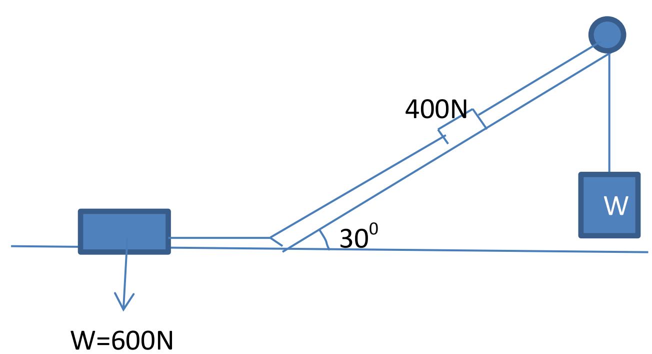
$\sum F_y = 0$

$F_{N1} = 600N$

$F_{f1} = \mu F_N$

$= 0.1 * 600$

$= 60N$



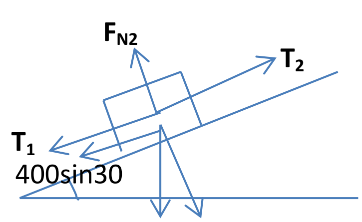
$\sum F_x = 0$

$T_1 - F_{f1} = 0$

$T_1 - F_{f1} = 0$

$T1 = 60N$

$\sum F_y = 0$



F_{N2} = 400cos30 346.4N

400N 400cos30

Ff = μF_{N2}

= 0.1 *346

= 34.64N

∑FX = 0

W – T_1 – F_{f2} – 400sin30 = 0

W – 60 – 34.64 – 200 = 0

W = 295N

Ex5.: a 400N block is resting on a rough horizontal surface as shown in fig. for which the coefficient of friction is 0.4 find the value of horizontal force **P** required to cause motion:

Sol.

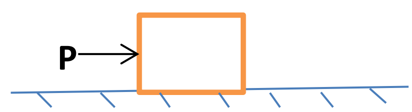
∑Fy = 0

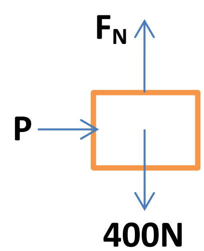
F_N = 400N

$F_f = \mu . F_N$

F_f = 0.4 * 400 160N

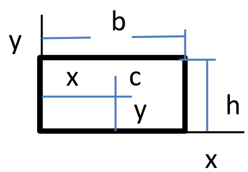
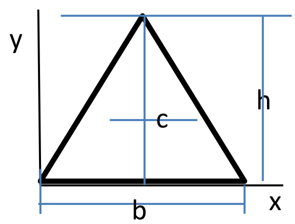
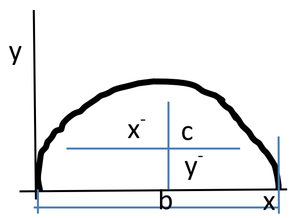
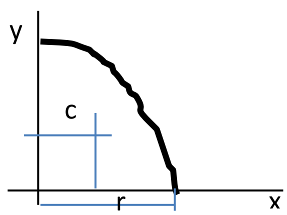
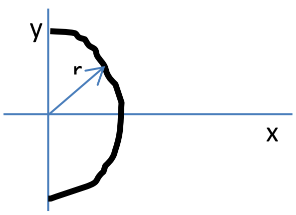
∑Fx = 0





Centroid : are center of gravity of areas and lines.

Centroid of common geometric shapes:

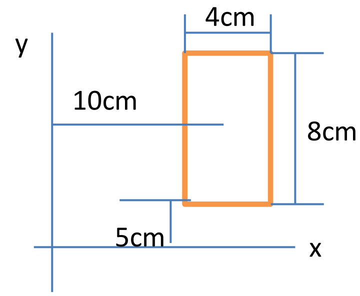
shapes	area	$\bar{x}$	$\bar{y}$
	**bh**	**b/2**	**h/2**
	**bh/2**	**b/2**	**h/3**
	$\Pi r^2/2$	**b/2 = r**	**4r/3Π**
	$\Pi r^2/4$	**0.424r**	**0.424r**
	$\Pi r^2/2$	**2r/Π**	**0**

Ex1.: for the following example find the centroid of the given shapes:

$\bar{x} = 10 + 2$

$= 12cm$

$\bar{y} = 5+4$

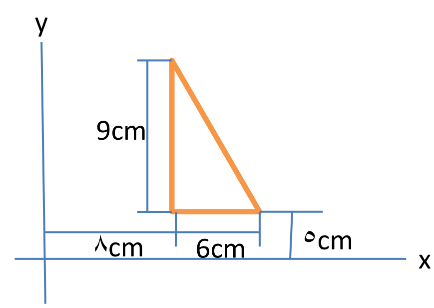


$\bar{X} = 8 + 6/2$

$= 10CM$

$\bar{Y} = 5 + 9/3$

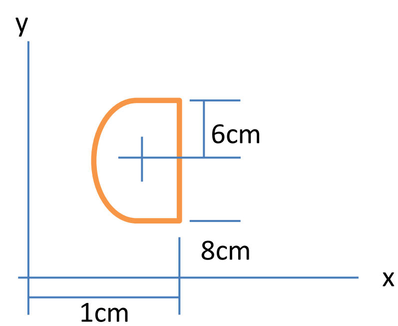
$= 8cm$



$\bar{x} = 15 – 0.24r$

$\bar{y} = 8 + 6$

$= 14cm$



Centroid of composite area:

Engineering area are composed of combination of simple shapes:

If a given area can be divided in to parts each centroid of which is known this equation could be used:

$\bar{x} = (A_1x_1 + A_2x_2 + A_3x_3 + \ldots\ldots Anxn)/\sum Ai$

$\bar{y} = (A_1y_1 + A_2y_2 + A_3y_3 + \ldots\ldots Anyn)/\sum Ai$

$\bar{x}$, $\bar{y}$ is the centroid of the whole area.

x_1 , x_2 ,…. Xn

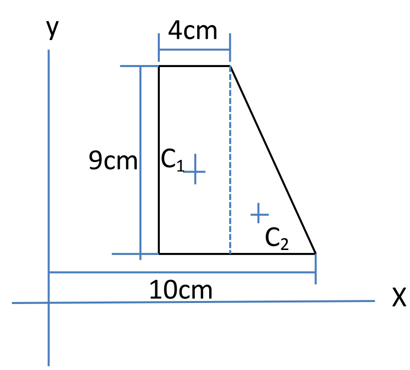
y_1 , y_2 ,…. Yn

Ai is the whole area.

Ex1.: find the centroid for the shape shown bellow:

$A_1 = 4 * 9 = 36$ cm

X1 = 2cm , y1 = 4.5cm



$A_2 = 6 *9/2$

$= 27cm^2$

$X_2 = 4 + 6/3$

= 6cm

Y2 = 9/3

= 3cm

$A = A_1 + A_2$

= 36 + 27

$= 63cm^2$

$X^{-} = (A_1X_1 + A_2X_2)/\sum Ai$

$= (36 * 2 + 27 * 6)/63$

$= 3.71cm$

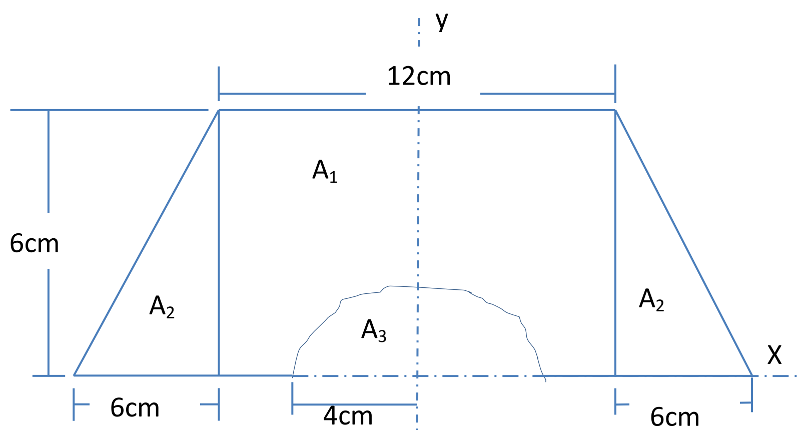
$y^{-} = (A_1Y_1 + A_2Y_2)/ \sum Ai$

$= (36 * 4.5 + 27 * 3)/63$

$= 3.85\ cm$

EX.2: find the centroid for the shaded area:

Sol.:



$A_1 = 12 * 6$

$= 72cm^2$

$A_2 =(3 * 6)/2$

$= 9cm^2$

$A_3 =(4^2\pi)/2$

$= 25cm^2$

$Ai = A_1 + A_2 + A_3$

$= 72 +2 *9 – 25$

$= 65cm^2$

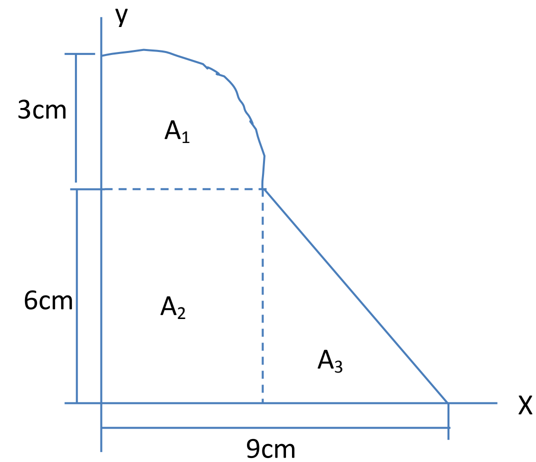
$y_1 = 3cm$, $y_2 = 6/3 = 2cm$, $y_3 = 0.424r$

$y^{-} = (A_1y_1+2A_2y_2- A_3y_3)/(A_1 +2A_2 –A_3)$

$= (72 * 3 + 2(9 *2) – 25 * 1)/65$

= 3.22cm , C(0 , 3.22)

EX.3: find the centroid for the area shown in fig.:



$A_1 = r_2\pi/4$

$= (3)2 *\pi/4$

$= 7cm_2$

$A_2 = 3 *6$

$= 18cm^2$

$A = A_1 + A_2 + A_3$

$= 7 + 18 + 18$

$= 43cm^2$

$X_1 = 0.424r$

$= 1.27cm^2$

$Y_1 = 6+ 0.424 *r$

$= 7.27cm$

$X_2 = 3/2 = 1.5$ cm , $Y_2 = 6/2 = 3cm$

$X_3 = 3 + 6/3 = 5cm$, $Y_3 = 6/3 = 2cm$

$X^{-} = (A_1X_1 + A_2X_2 + A_3X_3)/(A1 + A2 +A3)$

$= (7 * 1.27 + 18 *1.5 + 18 *5)/43$

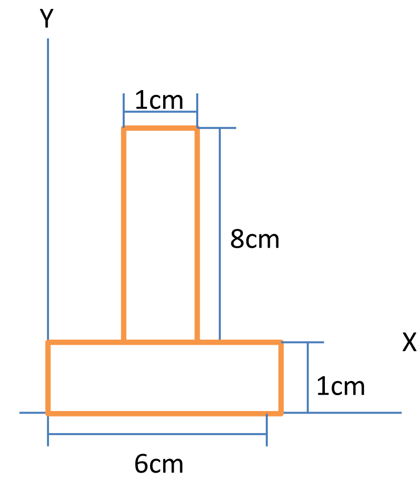
$= 2.9cm$

$y- = (A_1Y_1 + A_2Y_2 + A_3Y_3)/(A_1 + A_2 +A_3)$

$= (7 * 7.27 + 18 *3 + 18 *2)/43$

= C(2.9 , 3.27)

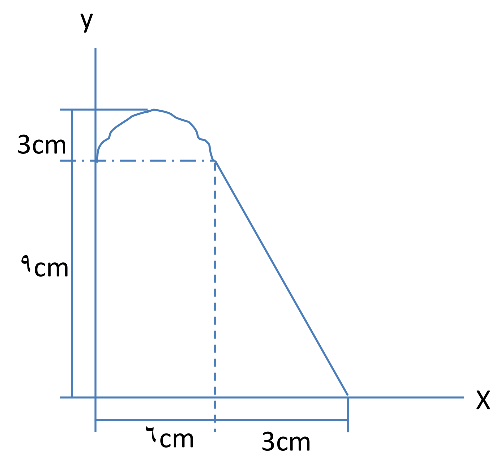
Ex.4: find the center of area of shape shown in fig.:



Ans. $\bar{X} = 3$

$\bar{Y} = 3.07$

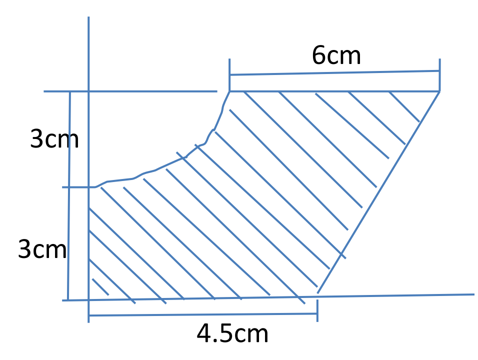
Ex.5: find the center of area of shape shown in fig.:



$\bar{X} = 3.7cm$

$\bar{Y} = 5.2cm$

Ex.٦: find the centroid of the shaded area?



Ans:.

X- = 4.5cm

Y- = 2.9cm

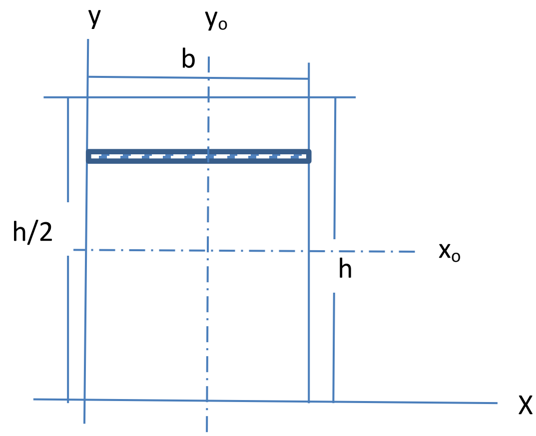
Moment of inertia (I):

$Ix = X_2 . dA$ (moment of inertia about X – axis)

$Iy = y_2 . dA$ (moment of inertia about Y – axis)

EX.: find moment of inertia for a rectangle of base (b) and depth (h) with respect to:

1- The x- axis
2- The y- axis



Ans.:

$$Ix = \int_{-h/2}^{h/2} (y^2).\, dA$$

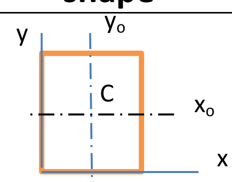
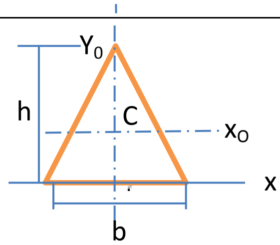
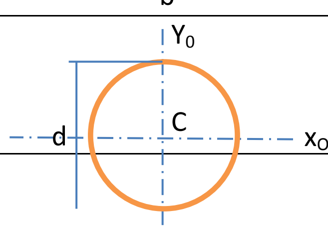
$$= \int_{-h/2}^{h/2} y^2 .\, b .\, dy$$

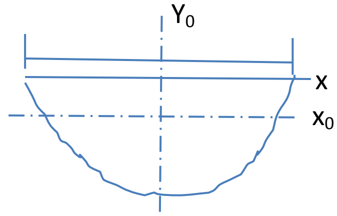
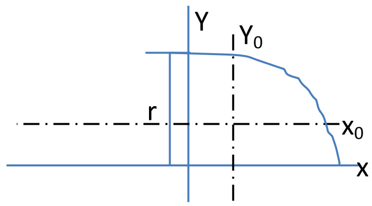
$$Ix = b(y^3)^{h/2}$$

$$= b/2(h^3/3 + h^3/3)$$

$$= bh^3/3$$

Moment of inertia for geometric shape:

shape	Ix	Iy
	**$bh^3/12$**	**$hb^3/12$**
	**$Bh^3/36$**	**$Hb^3/48$**
	**$\Pi d^4/64$**	**$\Pi d^2/64$**

	$\Pi r^2/4$	**$\Pi r^2/4$**
	**$0.007d^4$** **$0.11r^4$**	**$\Pi r^4/8$**
	**$0.055r^4$**	**$0.055r^4$**

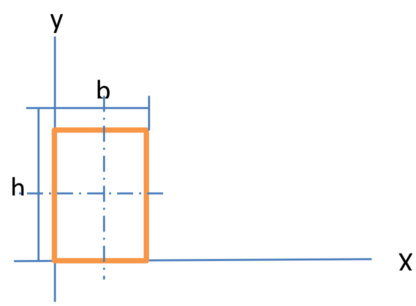
Transfer formula:

Some time its necessary to transfer the moment of inertia from one axis to another axis.

On the transfer formula:

$Ix = Ix_o + Ad^2$

Ex.1: for the following shapes find the moment of inertia about the x- axis:



$Ix_o = bh^3/12$

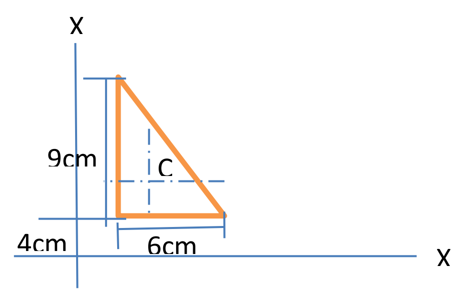
$= 256cm^4$

$Ix \ = Ix_o + Ad^2$

$= 6 * 8^3/12$

$= 256 + (8 * 6) * 4^2$

$= 102.4cm^2$



Ex.2: b = 6cm , h = 9cm

$Ixo = bh^3/36$

$= 6 * 9^3/36$

$= 127.5cm^4$

$Ix = Ix^0 + Ad^2$

$= 127.5 + (1/2 * 6 * *9)*(7)^2$

$= 1444.5 cm^2$

Ex.3: find I about y – axis if r 5cm:

$d = 0.424r$

$= 2.12cm$

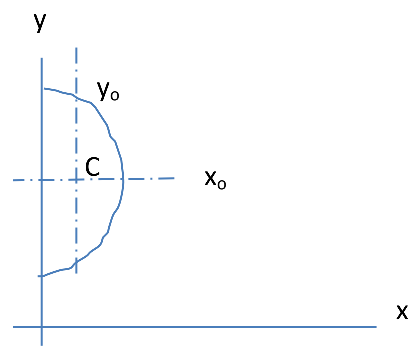
$Iyo = 0.11r^4$

$= 0.11(5)^4$

$= 68.7cm^4$

$Iy = Iy_o + Ad^2$

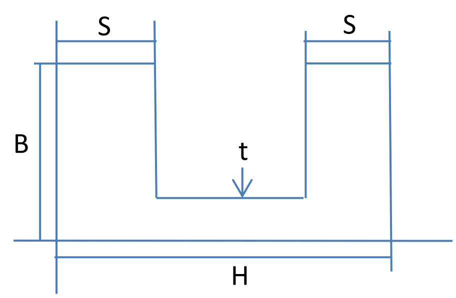
$= 68.7 + \pi r^4/2 * (2.12)^2$



Moment of inertia for composite areas:

When a composite area can be divided into geometric element (rectangle , triangle , …. Etc) for which the moment of inertia are known the moment of inertia for the separate elements.

Ex.1: find the moment of inertia about the z – axis:



S = 4mm , B = 8mm , t = 2mm , H = 16mm

$Iz = I_1z + I_2z + I_3z$ ……. (1)

$I_1z = I_3z$

$Iz = 2I_1z + I_2z$ ……… (2)

$I_1z = I_1x + d_1^2A$ …….. (3)

$I_2z = I_2x + d_2^2A_2$ …….(4)

Sub in (2):

$Iz = 2(I_{1X} + d_1^2A) + I_2X + d_2^2A_2$

$I_1x = SB_3/12$

$= 4(8)^3/12$

$d_1 = B/2\ 8/2 = 4$

$A_1 = B.S$

$= 4 * 8 = 32cm^2$

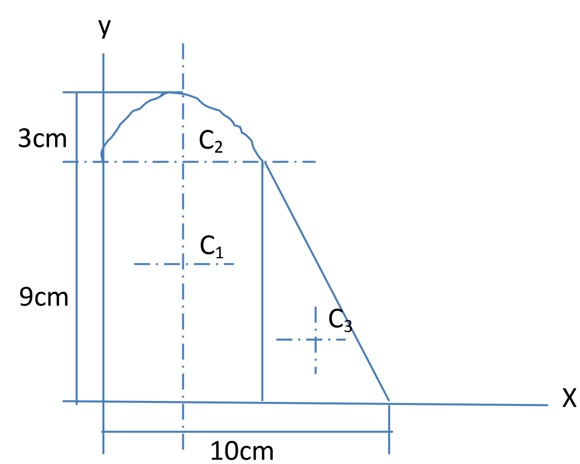
$I_2x = 8(2)^3/12$

$d_2 = 2/2 = 1$, $A_2 = 8 * 2 = 16$

$Iz = 1386.6mm4$

Ex.2: find moment of inertia for the composite area shown in fig. with respect to x – axis:

Ans. :



$A_1 = 6 * 9 = 54$

$d_1 = 4.5 - 2 = 2.5$

$Ix_1 = (6 * 9^3)/12$

$= 365cm^4$

Semicircle:

$A_2 = 3^2\pi/2 = 14.14cm^4$

$d_2 = 0.424 * 3 + 2$

$= 3.27cm$

$Ix_2 = 0.11r^4 = 0.11 * 3^4 = 8.91cm^4$

Triangle:

$A_3 = ½ * 4 * 9 = 18cm^2$

$d_3 = 6 – 2 = 4cm$

$Ix_3 = bh^3/36 = 4 * 9^3/36 = 81cm^4$

$Ix = (Ix_1 + A_1d_1^2) + (Ix_2 + Ad_2^2) + (Ix_3 + A_3d_3^2)$

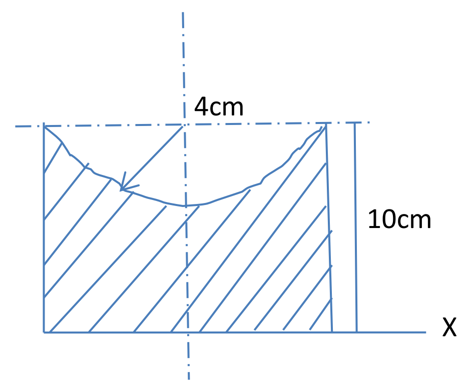
$Ix = 365 + 54 *(2.5)^2 + 8.91 + 14.14 * (3.27)^2 + 81 + 18 *4$

$= 1232cm^4$

Ex.3: find Ix for the shaded are:

$Ix = (8 *10^3/12 + 10 *8 *5^2) – 0.114^4 + 4^2\pi/2 (10 – 0.424 * 4)$

$= \ cm^4$



Dynamics:

Dynamics: is the branch of mechanics which studies the motion of bodies.

Velocity (v): is the rate of change of distance:

V = ds/dt

S = displacement

t = time

unit = m/sec

acceleration (a): we define acceleration as the time rate of change of velocity:

a dv/dt

unit = m/sec^2

linear motion:

in linear motion the particle travels a long straight line. if a body travel in a straight line from point A to B these formula can be used:

$\mathbf{v = v_0 + at}$

$\mathbf{s = v_0 t + 1/2 a t^2}$

$\mathbf{v^2 = v_0^2 + 2as}$

freely falling bodies or thrown up:

$\mathbf{v = v_0 + gt}$

$\mathbf{s = v_0 + ½ gt}$

$\mathbf{v^2 = v_0^2 + 2gs}$

Ex.1: a body moved from res with constant acceleration 4m/sec2 find:

1- Velocity after 10sec.
2- Distance after 8sec.
3- Distance when its velocity reaches 12m/sec.

   Sol.

   1- $V = v_0 + at$

   $V = 0 + 4 * 10$

   $= 40 m/sec$

   2- $S = v_0 + 1/24 * 8^2 = 128m$

   3- $V^2 = v_0 + 2as$

   4- $12^2 = 0 + 2 * 4 * s$

   $s = 144/8 = 18m$

Ex.2: a body is moving with an initial velocity and uniform acceleration of 0.03m/sec2 for 20sec moving mechanism is broken and the body travels 18m through 9sec find:

1- the initial velocity.
2- total displacement.

$V = s/t$

$= 18/9 = 2m/sec$

$V = v_0 + at$

$2 = v_0 + 0.03 * 20$

$V_0 = 1.4m/sec$

$S_1 = v_0 t + 1/2 at^2$

$= 1.4 * 20 + ½ * 0.03 * 20^2$

$= 34m$

$S = 34 + 18$

$= 52m$

Ex.2: a body is moving with an initial velocity and uniform acceleration of $0.03m/sec^2$ for 20sec the moving mechanism is broken and the body travels 18m through 9sec. find:

1- the initial velocity
2- total displacement

sol.

$V = s/t$

$= 18/9 = 2m/sec$

$V = v_0 + 0.03 * 20$

$V_0 = 1.4m/sec$

$S_1 = v_0 t + 1/2 at^2$

$= 1.4 * 20 + ½ * 0.03 * 20^2$

$= 34m$

$S = 34 + 18$

$= 52m$

Ex.3: a body is thrown upward with an initial velocity of 19.6m/sec calculate:

1- velocity after 1.5sec
2- time to reach maximum a height
3- time to reach maximum a height of 18.375m

Ans.

1- $V_0 = 19.6m/sec$

$V = v_0 + gt$

$V = 19.6 – 9.81 * 1.5$

$= 4.9m/sec$

2- V = 0 at max. height

$V = v_0 + gt$

$0 = 19.6 – 9.81 * t$

$t = 19.6/9.81$

$= 2sec$

3- $S = v_0 + ½ gt^2$

$18.375 = 19.6 *t + ½ * 9.81 *t^2$

$(9.81t^2 – 39.2t + 36.75 = 0)/2.45$

$4t2 – 16t + 15 = 0$

$(2t -3)(2t -5) = 0$

$t = 1.5sec , t = 2.5sec$

Ex.4: a car started motion from rest its speed up 40b m/sec with a constant acceleration of $4m/sec^2$ runs at this speed a times and finally comes to rest with acceleration of $5m/sec^2$ if the total distance travelled is 1000m. find the total time required:

Sol.

From A to B:

$V = v0 + at$

$40 = 0 + 4t$

$t = 10sec$

$s_1 = v_0t + 1/2at^2$

$= 0 + 1/2 * 4 *10^2$

$= 200m$

From B to C:

$S_1 + s_2 + s_3 = 1000$

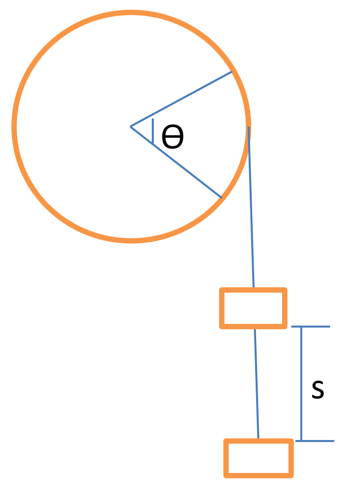
$200 + s_2 + 160 = 1000$

$S^2 = 640m$

$S_2 = v_0t + 1/2at^2$

$640 = 40t + ½ * 0* t^2$

$t = 16sec$, $t = 10 + 16 + 8 = 34sec$



Rational motion:

$S = r . ϴ$

$ds/dt = r\ dϴ/dt$

$ds/dt = v$ (linear speed in m/sec)

$dϴ/dt = w$ (angular speed in rad/sec)

$v = r . w$

$dv/dt = a_t$ (tangential acceleration in m/sec^2)

$dw/dt = \dot{α}$ (angular acceleration in rad/sec^2)

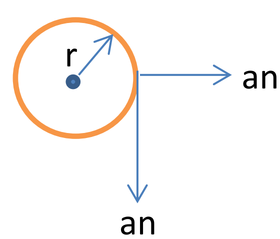
$at = r. \dot{α}$

$an = v2/r = r^2.w^2/r$

$an = rw^2$ (normal acceleration in rad/sec^2)

$a = √(an^2 + at2)$

a = total acceleration



relation ship between linear and rotation motion:

$s = r\ ϴ$

$v = r\ ω$

$a_t = r\ \dot{α}$

Linear motion	Rotation motion
$V = v_0 + a\ t$	$\omega = \omega_0 + \dot{\alpha}\ t$
$S = v_0 + ½\ a\ t^2$	$\theta = \omega_0\ t + 1/2\ \dot{\alpha}\ t^2$
$V^2 = v_0 2 + 2a\ s$	$\omega^2 = \omega_0^2 + 2\ \dot{\alpha}\ \theta$

$S = \theta$, $v = \omega$, $a = \dot{\alpha}$

Ex.1: a disc of radians 2m and angular speed of 3rad/sec and total acceleration 30m/sec^2. Calculate angular acceleration.

Sol.

$an = r\ \omega^2$

$an = 2 * 3^2$

$= 18m/sec$

$a^2 = a\ t^2 + a\ n^2$

$30^2 = a\ t^2 + 18^2$

$a_t = \sqrt{(24^2 + 18^2)}$

$= 24m/sec$

$at = r\ \dot{\alpha}$

$\dot{\alpha} = at\ /r$

$= 24/2$

$= 12$

Ex.2: a wheel of 0.5 m diameter tested on brake with a speed 60 km/hr when the brake is used the wheel stopped after 600m calculate:

1- ٢$\pi N = 600$

$N = 600/2 *3.14 * 0.25$

2- $S = r\ \theta$

$\theta = s/r$

$= 600/0.25$

$= 2400rad$

But $v_0 = r \omega_0$

$\omega_0 = v_0 / r$

$= 60 * 1000/60 * 60 * 0.25$

$= 400/6$ rad

$\omega^2 = \omega_0^2 - 2 \dot{\alpha} \theta$

$0 = \omega_0^2 - 2 \dot{\alpha} \theta$

$\dot{\alpha} = \omega_0^2 / 2 \theta$

$= (400/6)^2 / 2 * 2400$

$= 25/27 rad/sec^2$

Ex.3: point A moves in a circular path of 4m radians so that its arc distance from an initial position B is given by the relation: $S = 6t^3 – 4t$ where S in meters and t in sec. determine the tangential and normal component of the acceleration of the point for the instant when t = 2sec find also the total acceleration.

Ans.

$S = 6t^3 – 4t$

$V = ds/dt$

$at = dv/dt$

$v\ ds/dt = 18t^2$

$a_t = dv/dt = 36t$

a_t t = 2sec

$a_t = 36 * 2 = 72 m/sec^2$

$v = 18t^2 - 4$

$= 18 * 2^2 – 4 = 68 m/sec$

$a_n = v^2/r$

$= 68^2/4 = 1156 m/sec^2$

$a_{total} = \sqrt{(a_t^2 + a_n^2)}$

$= \sqrt{((1156)^2 + (72)^2)}$

$= 1158.2 m/sec^2$

Ex.4: the normal acceleration of a particle on the rim of pulley (4m) in a radius is constant at 3600m/sec2. Find the speed of the pullet in (r. p. m).

Sol.

$A_n = v^2/r$

$3600 = v^2/4$

V = 120m/sec

V = π(4 * 2)N/60

N = 120 *60/8 π

= 1834(r . p .m)

Strength of materials

Stress (δ): it is the ratio between the force that acts on a body to the perpendicular area of it.

δ = P/A

δ = stress (N/m^2)

P = force (N)

A = area (m^2)

Type of stress:

1- Shear stress
2- Tensile stress
3- Compression stress

Strain (ε): is done due to tensile or compressive force that cause the body to elongate or shorten.

$\varepsilon = \Delta L/L$

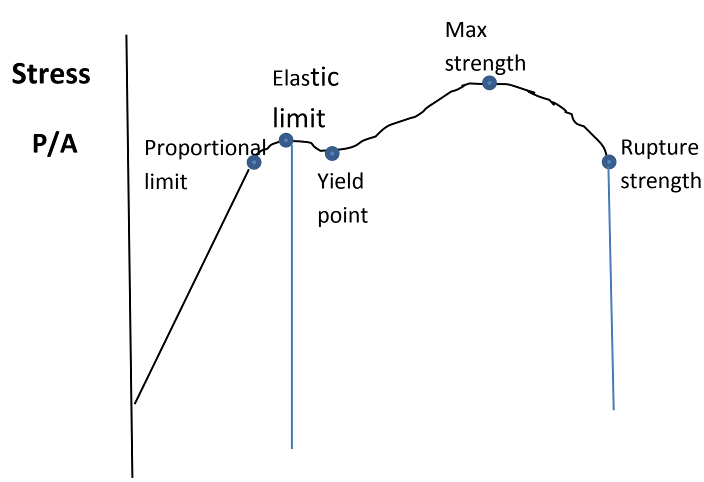
L = length

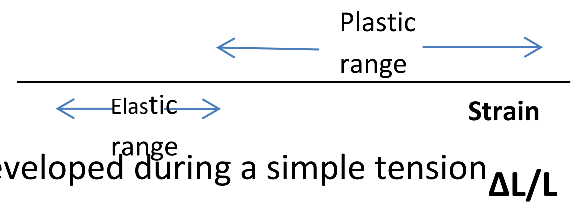
$\Delta L = L_2 - L_1$

$\varepsilon = L_1 - L_2/L$

$\Delta L = L_2 - L_1$

Stress – strain diagram:





Elastic limit: the maximum stress that may be developed during a simple tension test such that there is no deformation when the load removed.

For more structural materials the initial portion of stress - strain diagram is straight line which demonstrate Hooke's law that stress is directly proportional to strain the ratio is called modules of elasticity (E).

modules of elasticity (E): is the ratio between the stress to the strain:

$E = \delta / \varepsilon$

$E = (P/A) / (\Delta L/L)$

$\Delta L = PL/AE$

Elastic range: the heighest stress which resulting permanent deformation.

Plastic deformation: when the stress exceed the elastic limit the material does not return to its original shape upon release of load and becomes permanently deformed this deformation called plastic deformation.

Ex.1: A cylindrical bar 1m long elongates 2.54 mm under an axial tensile load find the strain in the bar:

Sol.

$\varepsilon = \Delta L/L$

$= 2.54/1000$

$= 2.54 * 10^{-3}$

EX.2: A bar 100cm long of cross-section 2cm * 2cm is acted upon a force 1000kg modulus of elasticity $E = 2 * 10^6$ kg/cm^2 . find the elongation in the bar and the stress and strain:

Sol.

$\Delta L = PL/AE$

$= 1000 * 100/4 * 2 * 10^6$

$= 0.0125cm$

$\delta = P/A$

$= 1000/4$

$250kg/cm^2$

$\varepsilon = \delta/E$

$= 250/2 * 10^6$

$= 0.000125$

Shearing stress (ĩ): a shearing stress is produced when ever the applied loads cause on section of a body to lend to slide past its section for example rivet resist shear across its cross-sectional area.

1- Single shear rivet:
   ĩ =P/A
2- Double shear:
   ĩ = P/Na
   $A = 2 * (\pi/4) * d^2$
   P = ĩ. An
   N = number of rivets

   EX.1: find the shear stress for the rivet shown in fig. its diameter 20mm and the load P = 30KN.
   ĩ = P/A
   $= 30 / (\pi/4)20^2$
   $= 0.0954KN/mm^2$

EX.2: A plate is fastened by three rivets arrange as shown in fig. the diameter of each rivet 20mm and the load P = 50KN find the shear stress developed.

ĩ = P/nA

$nA = 3 * (\pi/4) * d^2$

$= 9.424 * 10^{-4}\ m^2$

ĩ = P/nA

$= 50 * 10^3/9.424 * 10^{-3}$

$= 53..05 * 10^6\ N/m^2$

Torsional shearing stress:

ĩ = T r/J

T = Torque (Nm)

ĩ = shearing stress (N/m^2)

r = radius of shift (m)

J = polar moment of inertia

$J = \pi/32\ d^4$

$J_{for\ hallow\ shift} = \pi/32(D^{4} - d^{4})$

Ex.1: a torque of 16KN.m acts on 100mm dia. Shift find max. shearing stress in the shaft.

Sol.

ĩ = T r/J

$J = \pi d^2/32$

$= \pi(0.1)2/32$

$= 981.25 * 10^{-8} m^4$

$\tilde{\iota}$ = (16 * 1000 * 50/1000)/981.25 8 10^{-8}

= 81.5 * 10^{6} N/m^{2}

Ex.2: a torque of 9 * 10^{4} N.m acts on a hallow shaft the out side dia. Is 150mm and inside dia. 100mm determine the max. shearing stress in the shaft:

J = $\pi/32(150/100)^{4} - (100/1000)^{4}$

= $\pi/32(0.05)^{4}$

J = πr/J

= (9 * 104 * 50*1000)/ $\pi/32(0.05)^{4}$

<u>Angle of twist:</u>

T = GJΘ/L

Θ = TL/GJ

Θ = angle of twist (rad)

T = Torque transmitted (Nm)

L = length of shift (m)

J = polar moment of inertia (m^{4})

G= modulus of rigidity (N/m^{2})

Combined shaft

1- Free from one side
   $\Theta = \Theta_1 + \Theta_2$
   $\Theta = T1L_1/G_1J_1 + T_2L_2/G_2J_2$
   Θ_1 = angle of twist for steel
   Θ_2 = angle of twist for brass
2- Fixed from both sides:

$\Theta_1 = \Theta_2$

$T_1L_1/G_1J_1 = T_2L_2/G_2J_2$

$T = T_1 + T_2$

EX.1: what is the minimum diameter of a solid steel shaft that will not twist through mor than 3in in a 6m long when subjected to a torque 13.558Kn.m what max. shearing stress is developed:

Sol.

$\Theta = T\,L/GJ$

$\Theta = 3 * \pi/180$

$= 0.05236$ rad

$0.05236 = (13.558 * 10^3 * 6)/(82 * 10^9 * J)$

$J = 1.895 * 10^{-5}\ m^4$

$J = \pi/32\ d^4$

$d = 0.1178m$

$\tilde{\iota} = Tr/J$

$= 42.14 * 10^6 N/m^2$